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Can the String Scale be Related to the Cosmic Baryon Asymmetry?[1]

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ABSTRACT

In a previous work, a mechanism was presented by which Baryon asymmetry can be generated during inflation from elliptically polarized gravitons. Nonetheless, the mechanism only generated a realistic baryon asymmetry under special circumstances which requires an enhancement of the lepton number from an unspecified GUT. In this note we provide a stringy embedding of this mechanism demonstrating that if the model-independent axion is the source of the gravitational waves responsible for the lepton asymmetry, one can observationally constrain the string scale and coupling.

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1 Introduction

Inflation stands as the paradigm within which the problems of the standard big bang cosmology are resolved. Moreover, inflation provides a causal mechanism necessary for the formation of large scale structure (LSS) in the universe. Recently one of the present authors (SHSA) in a collaboration [1] presented a model in which the quantum fluctuations of gravity waves generated during the inflationary epoch can give rise to the cosmic matter-antimatter asymmetry. This mechanism relies on a rolling pseudo-scalar field coupled to a gravitational Chern-Simons term. However, a concrete model in the context of a fundamental theory was not specified in this work [1]. In the present note we provide an embedding of this leptogenesis mechanism into a 4D, $\mathcal{N} = 1$ SUGRA limit of closed type-I superstring or heterotic string theory, in which the model independent axion is dynamical. Since the axion is a universal field in any 4D, $\mathcal{N} = 1$ string theory compactification, we shall obtain a unique constraint on the string scale from a cosmological observable, the baryon asymmetry index.

1.1 Review of Inflationary Leptogenesis

The key to inflationary leptogenesis is the generation of a bi-refringent gravitational wave (BG) spectrum during the course of inflation. These BG waves are quantum mechanically produced during inflation if there is a non-vanishing axion correlated on the scale of the horizon at the beginning of inflation. Subsequently, through a triangle ABJ anomaly, a non vanishing quantum expectation value for $\mathcal{R}\wedge\mathcal{R}$ is generated from the contraction of BG waves which were sourced by the non-vanishing axion. When $< \mathcal{R} \wedge \mathcal{R} >$ integrated during the whole course of inflation, one gets a non-vanishing lepton number.

It is well known from the ABJ anomaly that CP violating interactions at one loop can induce an gravitational Chern-Simons term. The authors [1] proposed a model where this term can arise from a coupling of a pseudo-scalar to the gravitational Chern-Simons term. Our goal in this section is to briefly review this proposition by assuming such a term exists and finding explicit solutions of gravitational waves by linearizing the Einstein-Hilbert action. We will then proceed to review the APS [1] computation of the lepton number produced during inflation.

The authors [1] considered a general coupling between the the axion and the CP violating curvature invariant.

$$\mathcal{L}_{\rm int} \;=\; \mathrm{F}(\phi)\,\mathcal{R}^{\alpha}{}_{\sigma\mu\nu}\,\widetilde{\mathcal{R}}_{\alpha}{}^{\sigma\mu\nu}. \tag{1}$$

In terms of linearized perturbations up to second order this term is exactly[4].

$$\begin{aligned}\mathrm{Tr}(\mathcal{R}\wedge\mathcal{R}) = \tfrac{8i}{a^3}\Big[& \left(\tfrac{\partial^2}{\partial z^2}h_R\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_L\right) - \left(\tfrac{\partial^2}{\partial z^2}h_L\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_R\right) \\ & + a^2\left\{ \left(\tfrac{\partial^2}{\partial t^2}h_R\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_L\right) - \left(\tfrac{\partial^2}{\partial t^2}h_L\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_R\right) \right\} \\ & + \left(\tfrac{1}{2}\tfrac{\partial}{\partial t}a^2\right)\left\{ \left(\tfrac{\partial}{\partial t}h_R\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_L\right) - \left(\tfrac{\partial}{\partial t}h_L\right)\left(\tfrac{\partial^2}{\partial t\partial z}h_R\right) \right\}\Big] \quad ,\end{aligned} \tag{2}$$

where h_L and h_R are left and right helicities of the gravity waves respectively. The resulting gravity waves will be sourced by the axion via. the equations of motion

$$\begin{aligned}\Box h_L &= -\, i\, 2\tfrac{\Theta}{a}\,\dot{h}_L' \\ \Box h_R &= +\, i\, 2\tfrac{\Theta}{a}\,\dot{h}_R' \quad ,\end{aligned} \tag{3}$$

where

$$M_{pl}^2\Theta = 4(F''\dot{\phi}^2 + 2HF'\dot{\phi}). \tag{4}$$

The unspecified function F is constrained ultimately by its direct relationship with the observed baryon to entropy asymmetry

$$\tfrac{n}{s} \sim 6.5 \pm 0.4 \times 10^{-10} \tag{5}$$

By evaluating the the Green's functions of the gravitons eq (3) in conformal coordinates

$$\eta = 1/Ha = e^{-Ht}/H \ . \tag{6}$$

become

$$\left[\frac{d^2}{d\eta^2} - 2(\frac{1}{\eta} + k\Theta)\frac{d}{d\eta} + k^2\right] G_k(\eta,\eta') = i\frac{(H\eta)^2}{M_{Pl}^2}\delta(\eta-\eta'). \tag{7}$$

For $\Theta = 0$, the solution of this equation is

$$G_{k0}(\eta,\eta') = \begin{cases} (H^2/2k^3M_{Pl}^2)h_L^+(k,\eta)h_R^-(-k,\eta') & \eta < \eta' \\ (H^2/2k^3M_{Pl}^2)h_L^-(k,\eta)h_R^+(-k,\eta') & \eta' < \eta \ , \end{cases} \tag{8}$$

where h_L^- and h_R^+ are the negative and positive frequency solution to the gravity wave equation (3). For $\Theta = 0$, these solutions are the same as for h_L. After some more algebra we find the Greens function.

$$G_k = e^{-k\Theta\eta}G_{k0}e^{+k\Theta\eta'} \tag{9}$$

By contracting the Greens function we obtain the quantum expectation value of $\mathcal{R}\wedge\mathcal{R}$ The answer is:

$$\int d^3x < \mathcal{R}\widetilde{\mathcal{R}}(x) >= \frac{16}{a}\int \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3M_{Pl}^2}(k\eta)^2\cdot k^4\Theta \tag{10}$$

[4]In 2 the gravity waves are chosen to move in the z-direction with any loss of generality

As is well-known [2], the lepton number current, and also the total fermion number current, has a gravitational anomaly. Explicitly,

$$\partial_\mu J^\mu_\ell = \frac{1}{16\pi^2}\mathcal{R}\widetilde{\mathcal{R}} \tag{11}$$

where

$$J^\mu_\ell = \bar{\ell}_i\gamma^\mu\ell_i + \bar{\nu}_i\gamma^\mu\nu_i \ , \ \mathcal{R}\widetilde{\mathcal{R}} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\mathcal{R}_{\alpha\beta\rho\sigma}\mathcal{R}_{\gamma\delta}{}^{\rho\sigma} \ . \tag{12}$$

Inserting (10) into (11) and integrating over the time period of inflation, we find for the net lepton number density

$$n = \int_0^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3}\ \frac{1}{16\pi^2}\ \frac{8H^2k^3\eta^2\Theta}{M_{Pl}^2} \ . \tag{13}$$

The integral over k runs over all of momentum space, up to the scale μ at which our effective Lagrangian description breaks down. The dominant effect comes not from the usual modes outside the horizon at the end of inflation (super-horizon modes), $k/H < 1$, but rather from very short distances compared to these scales. The integral over η is dominated at large values of η, early times. The integral represents a compromise between two effects of inflation, first, to blow up distances and thus carry us to smaller physical momenta and, second, to dilute the generated lepton number through expansion. It is now clear that the dominant contribution to the right-hand side comes from $k\eta >> 1$, as we had anticipated. Performing the integrals, we find

$$n = \frac{1}{72\pi^4}\left(\frac{H}{M_{Pl}}\right)^2 \Theta H^3 \left(\frac{\mu}{H}\right)^6 \ . \tag{14}$$

We might interpret this result physically in the following way. The factor $(H/M_Pl)^2$ is the usual magnitude of the gravity wave power spectrum. The factor Θ gives the magnitude of effective CP violation and is governed by the dynamics of the theory at hand. The factor H^3 is the inverse horizon size at inflation; this gives the density n appropriate units. Finally, the factor $(\mu/H)^6$ gives the enhancement over one's first guess due to our use of strongly quantum, short distance fluctuations to generate $< \mathcal{R}\widetilde{\mathcal{R}} >$, rather than the super-horizon modes which effectively behave classically.

The crucial assumption in the above leptogenesis mechanism was the origin and the specific form of the function $F(\phi)$. While the authors argued that such a function can be generic in the low energy effective action of string theory, it was not specified. From another perspective one can ask a related question. Is the axion field in the low energy 4D effective action of string theory constrained to have couplings of the above form and if so what are the constraints on its dynamics? We shall discover that not only is such a term quite universal, but the the axion is required to be dynamical due to a very peculiar set of superspace constraints, namely the 'beta-function-favored constraints' βFCC[3].

2 Level Crossing from Inflationary Gravity Waves

The net lepton number that was calculated in [1] relied on the Atyiah-Singer (AS) index theorem. The index theorem relies on Manifolds with definite topology. In the inflationary slicing of De Sitter space, the topology is indefinite so applying the Atyiah-Singer index theorem should be handled with care. One way of addressing the validity of the lepton number calculation is to perform a level crossing analysis of fermions in the background of the gravitational waves. Our goal in this section is to show how the gravity waves may give rise to anomalous production of fermions via level crossing. In particular, if we have gravity waves which evolve in such a way that $R\tilde{R}$ has a non-vanishing integral, as is the case in our model, the fermion energy levels can cross from a negative to positive energy spectrum. This would explicitly establish the relation between fermion number generated via the presence of gravity waves rather than the usual topological arguments. In fact this is another way of obtaining the gravitational ABJ anomaly result.

The idea is that instead of performing the (gravity) triangle loop integral, one may focus on the Dirac equation in the background of the gravity waves and directly find the energy levels and how they evolve in time.

We begin by the action in which the (chiral) spin 1/2 fermion is covariantly coupled to gravity:

$$L_f = (\det\ e)(\bar{\Psi}_L i\gamma^\mu \nabla_\mu \Psi_L) \tag{15}$$

where

$$\nabla_\mu = \partial_\mu + \tfrac{1}{4}\omega_\mu^{ab}\gamma_{ab} \tag{16}$$

and ω_μ^{ab} is the spin-connection. We will use Greek indices for the space-time and Latin indices for the tangent space and as usual vierbeins e_μ^a relate these two and

$$\eta_{ab}e_\mu^a e_\nu^b = g_{\mu\nu} \tag{17}$$

The spin-connection can be solved in terms of vierbeins using the identity

$$D_\mu e_\nu^a = \partial_\mu e_\nu^a - \Gamma^\rho_{\mu\nu}e_\rho^a + \omega_\mu^{ab}e_{\nu b} \equiv 0 \tag{18}$$

where $\Gamma^\rho_{\mu\nu}$ is the Christoffel symbol.

Noting that $\gamma_\mu = e_\mu^a\gamma_a$ and the fact that

$$\gamma^a\gamma^{bc} = \tfrac{i}{2}(\eta^{ab}\gamma^c - \eta^{ac}\gamma^b) - \epsilon^{abcd}\gamma^5\gamma^d \tag{19}$$

we obtain

$$L_f = (\det\ e)(\bar{\Psi}_L i\gamma^\mu\partial_\mu\Psi_L + \mathcal{L}_{int}) \tag{20}$$

with

$$\mathcal{L}_{int} = -\tfrac{1}{4}\bar{\Psi}_L(\omega_a\gamma^a + i\tilde{\omega}_a\gamma^5\gamma^a)\Psi_L + c.c. \tag{21}$$

where

$$\omega_a = e^\mu_a\omega^{ab}_\mu\ , \quad \tilde{\omega}_d = \epsilon^{abcd}e^\mu_a\omega^{bc}_\mu. \tag{22}$$

For the chiral fermions $\gamma^5\Psi_L = \Psi_L$ and hence

$$\mathcal{L}_{int} = \tfrac{1}{4}\left[\bar{\Psi}_L(\omega_a + i\tilde{\omega}_a)\gamma^a\Psi_L\right] + c.c. \tag{23}$$

For our case the metric can be parameterized as

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right] \tag{24}$$

where h_{ij} is a symmetric, traceless and divergence-free tensor parameterizing the gravity waves. For the above metric, the spin connection can be written as

$$\omega^{ab}_\mu = \left(\omega^{ab}_\mu\right)_0 + \left(\omega^{ab}_\mu\right)_h \tag{25}$$

where $\left(\omega^{ab}_\mu\right)_0$ is the contribution to spin connection from the FRW metric (i.e. when $h_{ij} = 0$) and $\left(\omega^{ab}_\mu\right)_h$ is the contribution from the gravity waves. Here we will only work in the first order in h's, and hence $\left(\omega^{ab}_\mu\right)_h$ is proportional to the first order derivatives of h's.

In order to study the fermion level crossing, we study the Dirac equation:

$$\gamma^\mu\nabla_\mu\psi = \gamma^\mu(\partial_\mu + \tfrac{1}{4}\omega^{ab}_\mu\gamma^{ab})\psi = 0 \tag{26}$$

Next we note that the Dirac equation for the massless fermions is conformally invariant, i.e. for any given solution to the background $g_{\mu\nu}$, $(\psi, g_{\mu\nu})$, $\lambda^{\frac{d-1}{2}}\psi$ is also a solution to the Dirac equation on the background with metric $\lambda^{-2}g_{\mu\nu}$, e.g. see [4].[5] Hence we can simply focus on the gravity waves in the flat background, and then multiply the solution to the corresponding Dirac equation by $a^{3/2}$.

Let us now specifically solve the eigenvalue problem from the above Dirac equation.

$$(\gamma^i k_i + \omega^{ab}_\mu\gamma^\mu[\gamma_a, \gamma_b])\Psi_D = \gamma^0 E\Psi_D \tag{27}$$

Since we are studying the phenomenon of level crossing we are in the adiabatic regime of the quantum mechanics. If the eigenvalues adiabatically cross one another, then fermion creation is established and we have level crossing across the Dirac sea. We

[5]This statement is not valid in the region (or locus) where $\partial_\mu \ln\lambda$ is vanishing.

will aim to find the eigenvalues of the dirac equation in the gravity wave background during the complete inflationary epoch. From equation (17) in our paper, we have the solution for the background gravitational wave :

$$h_L = e^{ikz} \cdot (-ik\eta) e^{k\Theta\eta} g(\eta) \tag{28}$$

where

$$g(\eta) = \exp[ik(1-\Theta^2)^{1/2}\eta(1+\alpha(\eta))] \ , \tag{29}$$

where $\alpha(\eta) \sim \log\eta/\eta$. And the right handed component of the gravity wave can be obtained by changing the sign of Θ in h_L.

We can solve explicitly for the non vanishing components of the spin connection from eq (18). For our specific situation we will consider the non-vanishing time-like component of the spin connections. In other words we will choose a gauge where both the fermion and gravity plane waves are moving in the z-direction, without loss of generality. We find for the spin connection after much of algebra,

$$\omega_0^{12} = -i(h'_L - h'_R)/2 \tag{30}$$

where $'$ denotes a derivative with respect to conformal time. Plugging (28) into (27) we get

$$\gamma^3 \left[k + k\gamma^5 \left(-k\Theta\eta\cosh(k\Theta\eta) + (1-ik\eta)\sinh(k\Theta\eta)\right)\right]\Psi_D = \gamma_0 E\Psi_D \tag{31}$$

in our mechanism to get successful leptogenesis $\Theta \ll 1$, and also we discussed in our paper the main contribution comes from the region $k\eta \gg 1$ while $k\eta\Theta \ll 1$. So in the leading order the above eigenvalue equation takes the form

$$\gamma^3 k\left[1 - i\gamma^5(k\Theta\eta)k\eta\right]\Psi_D = \gamma_0 E\Psi_D \tag{32}$$

and hence lepton number production when

$$(k\Theta\eta)k\eta \to 1 \tag{33}$$

which is of course possible within the ranges of $k\eta \sim 10^3 - 10^4$ and $\Theta \sim 10^{-8} - 10^{-9}$ that is considered in [1], in accord with the recent cosmological data. We stress that this condition is exactly the regime considered in [1] and is consistent with lepton number production via computing the quantum expectation value of $\langle R \wedge R \rangle$ (provided that $\Theta \ll 1$). The authors of [1] found the Greens functions for the modes at short distances, $k\eta \gg 1$ which dominated lepton number. It is indeed encouraging that the level crossing analysis yields the same condition for lepton number production. [6]

[6] In the Ref. [4], the computations has been done for FRW background in the "global" coordinates, the coordinates in which the metric is conformal to Einstein static Universe, $R \times S^3$.

3 A Supergravity Realization

We begin our analysis from the compactification of the heterotic string to its 4D, $\mathcal{N} = 1$ supergravity limit. For concreteness we consider the compactification to be on a Calabi-Yau 3-fold or a deformed conifold. In order to discuss cosmology in this context it is important to stabilize all moduli since they can lead to disasterous relic overproduction in the early universe since this will be inconsistent with the bounds of nucleosynthesis. Some attempts in the heterotic compactification to 4D has been made with the inclusion of fractional fluxes on a 3-cycle embedded the CY_3-fold [6]. We will assume that all moduli except the axion are stabilized.

Our starting point is the 10D Heterotic string action in Einstein frame [5].

$$S = \int d^{10}x\sqrt{g_{10}}\Big(\mathcal{R} - \frac{1}{2}\partial_A\phi\partial^A\phi - \frac{1}{12}e^{-\phi}\,H_{ABC}\ -\ \frac{1}{4}e^{\frac{-\phi}{2}}tr(F_{AB}F^{AB})\ \Big) \tag{34}$$

where

$$H_3 = dB_2 - \frac{1}{4}(\Omega_3(A) - \alpha'\Omega_3(\omega)) \tag{35}$$

where $\Omega_3(A)$ and $\Omega_3(\omega)$ are the gauge and gravitational Chern-Simons three-forms respectively.

$$\Omega_3(A) = Tr(dA \wedge A + \frac{2}{3}A \wedge A \wedge A) \tag{36}$$

We now dimensionally reduce the 10D action to 4D, $\mathcal{N} = 1$ supergravity coupled to the $SO(32)$ or $E_8 \times E_8$ gauge sectors by choosing a four-dimensional Einstein frame metric , $g^S_{MN} = g^E_{MN}e^{\frac{\phi}{2}}$.

$$ds^2_{10} = e^{-6\sigma}ds^2_4 + e^{2\sigma}g_{mn}dy^m dy^n \tag{37}$$

where g_{mn} is a fixed metric if the internal dimensions normalized to have volume $4\alpha'^3$

$$S_{4D} = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}[\mathcal{R}\ -\ \frac{2\partial_\mu S^*\partial^\mu S}{(S+S^*)^2} - \frac{1}{2}G^{i\bar{j}}G^{k\bar{l}}\partial_\mu T_{i\bar{l}}\partial^\mu T_{\bar{j}k}] \tag{38}$$

The field $T_{i\bar{l}}$ are associated with moduli in the internal dimensions. For our purposes we are assuming moduli stabilization so these fields will not be relevant. For a further discussion of this issue in specific moduli stabilization scheme we refer the reader to [6]

It is now useful to relate the string scale and coupling to the four dimensional moduli. The four dimensional gauge coupling is

$$g^2_{YM} = e^{\psi}. \tag{39}$$

where the four dimensional dilaton ψ is related to the ten-dimensional dilaton and volume modulus via

$$\psi = \frac{\phi}{2} - 6\sigma \tag{40}$$

Also related to the ten dimensional dilaton is the volume scalar ρ.

$$\rho = \frac{\phi}{2} + 2\sigma \tag{41}$$

The fields ψ, ρ are related to the scalar components of the two [7] $\mathcal{N} = 1$ chiral superfield S

$$S = e^{-\psi} + ia \tag{42}$$

where a is the model independent axion field which arise from the spacetime and internal components of B_{AB} respectively. Specifically,

$$(*da)_{\mu\nu\rho} = e^{-2\psi} H_{\mu\nu\rho} \tag{43}$$

The heterotic string compactified to 4 dimensions can exhibit some moduli stabilization due to fluxes, hence we refer the reader to the work of Gukov et. al. [6]. In what follows, we focus our attention on the axionic sector of the the 4D heterotic string. The bosonic low energy effective action will take the form

$$S_{4d} = S_{gravity} + S_{axion} + S_{CS} \tag{44}$$

$$S_{4d} = \frac{2}{\alpha'} \int d^4x \sqrt{-g} (\mathcal{R}_4 + \frac{1}{2} \partial_\mu a \, \partial^\mu a + V(a)) \tag{45}$$

where $V(a)$ is the potential for the axion which will be responsible for inflation. We shall discuss the form of this potential in the following section. Finally, there will be an important contribution from the Green-Schwartz mechanism [11]

$$\int d^4x F(a) \mathcal{R} \wedge \mathcal{R} \tag{46}$$

where

$$F(a) = a \mathcal{V} \mathrm{M}_{4\mathrm{pl}} \alpha' \tag{47}$$

where $\mathcal{V}$ is a volume factor measured in string units and is determined on the dimensionality of the compactification. Take note that eq (46) is precisely the term which is needed for our inflationary leptogenesis to occur. Most importantly this interaction and its coupling is universal.

[7] Due to our choice of metric ansatz the gravitational coupling is $\kappa_4^2 = \frac{\alpha'}{4}$. The physical gravitational coupling differs from this by a constant rescaling.

3.1 Natural Heterotic Inflation

In the previous section we demonstrated that a 4D effective description of the heterotic string can have a dynamic axion. In the presence of potential which satisfies the flatness conditions for inflation, the axion can act as the inflaton field. In general, axions acquire oscillatory potentials reflecting the periodicity of the Θ vacua. These sorts of potentials were first discussed as inflationary potentials by Freese et. al and extended to Supergravity by Kawasaki et. al under the name "Natural Inflation". These potentials are constrained from the superpotential in $\mathcal{N}=1$ supergravity and are naturally flat due to the shift symmetry in the chiral superfield. In particular, the F-term potential in Planck units is

$$V_F = e^K\left(\sum_{i,j} K^{i,j} D_i W D_j W - 3|W|^2\right), \tag{48}$$

where i,j runs over all moduli fields, K is the Kahler potential for T and S, $K^{i\bar{j}}$ is the inverse of $\partial_i\partial_{\bar{j}}K$. These authors [8, 9] found flat potentials which could generate inflation. Recently, Blanco-Pillado et. al. have demonstrated a general class of axionic potentials which generate inflation as an extension to the KKLT mechanism[7, 10]. It will be interesting to realize a similar scenario in our heterotic example.

It is not necessary that the axion be identified with the inflaton field, however. Inflation could be driven by another field and the axion, through its equation of motion in this background can track the slow rolling of the inflaton field.

$$\ddot{a} + 3H_{inf}\dot{a} = -V'(a) \tag{49}$$

where H_{inf} is the Hubble parameter associated with the inflaton field rather than the axion. Since our mechanism only requires the axion to be coherent over the Hubble scale during the course of inflation, this amounts to it having a small slow roll paramenter, $\epsilon = \frac{1}{2}(\frac{\dot{a}}{H_{inf}M_{pl}})^2 << 1$. This equation arises from the equation of motion of the axion during inflationary expansion. In other words the axion may have a flat potential to guarantee its slow rolling but not necessarily have enough energy density to yield enough inflation. The important point for our analysis is that we have inflation and its associated scale since our calculation involves constraining the Hubble constant during inflation from recent CMB measurements.

4 An Observational Constraint on the String Scale

We are now ready to determine the string length scale and coupling by relating it to observed baryon to entropy ratio. The crucial point is that any four dimensional

compactification of string theory is endowed with the model independent axion which couples to $\mathcal{R}\wedge\mathcal{R}$ via the Green-Schwarz mechanism. It is through this fact that we are able to uniquely observationally constrain fundamental string parameters. Recall that the baryon to entropy ratio is fully determined by the gravitational power spectrum, the CP-violation term which enhances the amplitude of the gravitational waves and the momentum dependence of the gravitational waves. The CP-violation term Θ is fully controlled by the dynamics of the fundamental theory. We shall obtain a result which constrains the ratio between M_{pl} and M_{10}, the fundamental and four dimensional Planck scale, respectively, through the observed baryon to entropy ratio.

In the previous section we motivated a heterotic extension of inflationary leptogenesis via. gravitational waves which experience birefringence during inflation. The baryon to entropy ratio was calculated and determined in terms of three terms, each signifying distinct physics.

$$\frac{n}{s} = 10^{-6}\Theta(\frac{H}{M_{pl}})^{\frac{7}{2}}(\frac{\mu}{H})^6 \tag{50}$$

where $\frac{H}{M_{pl}}$ is constrained by WMAP to have an upper bound of $\frac{H}{M_{pl}} \sim 10^{-4}$. Recall that we can express $F(a)$ from the equation of motion of the gravitational waves in terms of Θ.

$$M_{pl}^2\Theta = 4(F''\dot{\phi}^2 + 2HF'\dot{\phi})\mathcal{V}. \tag{51}$$

Because the axion is slowly rolling during inflation and is linearly coupled to the Chern-Simons form, the first term vanishes and we obtain.

$$\Theta = 4F'H\dot{a}\frac{1}{M_{pl}^2}\mathcal{V} \tag{52}$$

This term is completely determined by string theory since it involves the model independent axion. Therefore, the string scale and coupling will determine the observed baryon to entropy ratio. We can simplify Theta by using the relation between the slow roll parameter and the velocity of the axion field. After a little algebra we get,

$$\Theta = (\frac{H}{M_{pl}})^2\mathcal{V}\sqrt{\epsilon}. \tag{53}$$

Notice that $\mathcal{V}$ the volume factor can lead to an enhancement in the Greens function of the gravity waves. This could be important for future CMB observations of gravity waves, since one could obtain an enhancement in the B-mode polarization in the gravity wave power spectrum. We are currently investigating this possibility by

extending the work of Lue et. al [12, 14] Specifically, $\mathcal{V} = \frac{\mathrm{M_{pl}}}{\mathrm{M_{10}}}(\mathrm{VolM_{10}^6})$ and it is useful to recall that

$$M_{pl}^2 = M_{10}^8 Vol \tag{54}$$

and

$$M_{10}^8 = \alpha'^{-4} g_s^2 \tag{55}$$

where Vol is the volume of the Calabi-Yau 3-fold. Hence,

$$\mathcal{V} = (\frac{\mathrm{M_{pl}}}{\mathrm{M_{10}}})^3 \tag{56}$$

with the above definitions eq (50) become

$$\frac{n}{s} = 10^{-6}(\frac{H}{M_{pl}})^{1/2}(\frac{\mu}{H})^6(\frac{M_{pl}}{M_{10}})^3 \tag{57}$$

We can set the cutoff scale of the Chern-Simons interaction to be the string scale, $\mu = M_{10}$ and and demanding from observations that

$$\frac{n}{s} \sim 10^{-10} \tag{58}$$

we get finally

$$\frac{M_{10}}{M_{Pl}} \sim 10^{-2} \tag{59}$$

which sets the string scale to be

$$\frac{g_s^{1/4}}{M_{pl} l_s} \sim 10^{-2} \tag{60}$$

We immediately get a lower bound for the string scale to be aroun $10^{17} GeV$ provided that the string coupling is order unity. Clearly as we weaken the string coupling the string scale goes down.

5 Conclusion

In this work we have provided a stringy embedding of the inflationary leptogenesis mechanism of [1]. This was possible due to the universality of the coupling of the model independent axion to the Chern-Simons form in four dimensional compactifications of string theory. In the original work of [1] the baryon to entropy ratio required an unspecified enhancement to accommodate the observed value of $\frac{n}{s} \sim 10^{-10}$. In our realization this enhancement arose naturally due to the volume factor in eq (53). We chose to explicitly study the heterotic string theory for concreteness and found that

for reasonable values of the string coupling and length scale the observed baryon to entropy ratio can be generated during a period of inflation.

We have presented a fairly conservative compactification. If the volume of compactification is larger than the string scale, such as in warped compactifications, then the enhancement of the baryon asymmetry will be improved. It will be interesting to further investigate this mechanism in the context of brane world scenarios and stringy inflationary mechanisms driven by axions which were already studied by Kallosh et al and Blanco-Pillado et al.

In this leptogenesis mechanism the UV modes of gravitational waves coupled to volume enhancement factor $\mathcal{V}$ are responsible for the enhancement of the lepton number density. In future CMB experiments such as PLANK the "smoking gun" of inflation, the tensor to scalar ratio $r = \frac{T}{S}$ will be constrained for models with high scale inflation[13]. Likewise the tensor power-spectrum can be enhanced by the volume factor from string theory. In is intriguing to relate this mechanism directly to the the IR (superhorizon) gravitational power specturm associated with the CMB. We expect to report on this connection in a future paper [15]

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"Hidden deep in the heart of strange new elements are secrets beyond human understanding, new powers, new dimensions – world within worlds unknown."

Quote from Outer Limit Episode: "The Production and Decay of Strange Particles"

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